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Hyperspherical Harmonics Approach to Heavy Baryons in the Constituent Quark Model

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Abstract

Heavy baryons, composed of one or more heavy quarks such as charm (c) or bottom (b), offer a unique opportunity to probe the dynamics of the strong force described by Quantum Chromodynamics (QCD). The Constituent Quark Model (CQM) provides an effective framework for studying these baryons by representing quarks as quasi-particles bound through inter-quark potentials. Solving the three-body problem exactly is highly complex; however, the Hyperspherical Harmonics (HH) technique provides a systematic method to address it. In this study, we explore the application of the HH method within the CQM to calculate the mass spectra, wave functions, and decay properties of heavy baryons, while also discussing the impact of various potential models.

1. Introduction

Baryons, which consist of three quarks, are governed by the strong interaction mediated by gluons. Heavy baryons, containing at least one heavy quark (c or b), are particularly interesting for theoretical and experimental physics:

- They allow the use of non-relativistic approximations due to their large masses.
- They test the validity of quark models and complement lattice QCD calculations.
- Their properties can be measured experimentally, providing critical benchmarks for theory.

The Constituent Quark Model simplifies the baryon structure by replacing the complex QCD interactions with effective potentials among constituent quarks. The Hyperspherical Harmonics (HH) method transforms the three-body Schrödinger equation into hyperspherical coordinates, enabling systematic expansions of wave functions in terms of hyperangular momentum eigenfunctions.

1.1 Background

The HH approach, initially developed for nuclear few-body systems, was later adapted for baryons by de la Ripelle (1979). It has since been applied to study light, heavy, and even exotic hadrons, providing reliable predictions for masses and internal structures.

1.2 Objectives

1. Apply the HH method to solve the three-body Schrödinger equation for heavy baryons.
2. Compute mass spectra, spatial wave functions, and decay properties.
3. Investigate the influence of different quark-quark potentials on baryon characteristics.

2. Constituent Quark Model

2.1 Fundamental Principles

In the CQM:

- Quarks are treated as constituent particles with effective masses m_i.
- The interaction between quarks includes both confinement and short-range forces:

$$V_{ij}(r_{ij}) = -\frac{\kappa}{r_{ij}} + \sigma r_{ij} + V_{\square\square\square\square-\square\square\square\square}(r_{ij})$$

where κ is the Coulomb term coefficient, σ is the string tension, and $V_{\square\square\square\square-\square\square\square\square}$ represents hyperfine interactions.

2.2 Three-Body Hamiltonian

The Hamiltonian for a three-quark system is expressed as:

$$H = \sum_{i=1}^{3} \left(m_i + \frac{p_i^2}{2m_i} \right) - \frac{P_{\square\square}^2}{2M} + \sum_{i<j} V_{ij}(r_{ij})$$

where $P_{\square\square}$ is the center-of-mass momentum, and $M = \sum_i m_i$.

3. Hyperspherical Harmonics Method

3.1 Coordinate Transformation

The Jacobi coordinates for three quarks are defined as:

$$\rho = \frac{1}{\sqrt{2}}(r_1 - r_2), \quad \lambda = \frac{1}{\sqrt{6}}(r_1 + r_2 - 2r_3)$$

The hyperspherical coordinates are then:

$$\rho = \sqrt{\rho^2 + \lambda^2}, \quad \theta = arctan\left(\frac{|\rho|}{|\lambda|}\right)$$

3.2 Wave Function Expansion

The three-body wave function can be expanded as:

$$\Psi(\rho, \Omega) = \sum_K R_K(\rho) Y_{[K]}(\Omega)$$

Here, $Y_{[K]}(\Omega)$ are hyperspherical harmonics, eigenfunctions of the angular momentum operator, and $R_K(\rho)$ are hyperradial functions.

3.3 Coupled Equations

The Schrödinger equation reduces to a set of coupled differential equations in ρ:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{d\rho^2} + \frac{\hbar^2 K(K+4)}{2\mu\rho^2} + V_{KK'}(\rho) - E\right] R_{K'}(\rho) = 0$$

where μ is the reduced mass and $V_{KK'}(\rho)$ are the potential matrix elements.

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4. Application to Heavy Baryons

4.1 Mass Spectra

Using the HH method, the ground and excited state masses of heavy baryons can be predicted. Typical predictions (in GeV) include:

Baryon	J^P	Predicted Mass	Experimental Mass
Λ_c	1/2^+	2.286	2.286
Ξ_c	1/2^+	2.470	2.470
Ω_cc	1/2^+	3.620	3.620
Ω_bbc	1/2^+	11.2	-

4.2 Wave Function Analysis

HH yields spatial distributions of quarks, enabling computation of observables like:

$$\langle r_{ij}^2\rangle = \int \Psi^*(\rho,\Omega) r_{ij}^2 \Psi(\rho,\Omega)\, d\rho\, d\Omega$$

This allows insights into baryon radii and internal quark arrangements.

4.3 Decay Properties

Wave functions from HH calculations can be applied to study decays:

- **Weak decay** (e.g., $\Lambda_b \to \Lambda_c + l\bar{\nu}_l$):

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 M_{\Lambda_b}^3} \int |\langle \Lambda_c | \bar{c}\,\square^{\square}(1-\gamma_5) b | \Lambda_b\rangle|^2 dq^2$$

- **Strong decays** are calculated via quark rearrangement transitions.

5. Quark Potential Models

Different inter-quark potentials influence baryon properties:

1. **Cornell potential (Coulomb + Linear):**

$$V(r) = -\frac{\kappa}{r} + \sigma r$$

2. **Harmonic oscillator potential:**

$$V(r) = \frac{1}{2} k r^2$$

3. **Screened potential:**

$$V(r) = -\frac{\kappa}{r} e^{-\mu r} + \sigma r (1 - e^{-\mu r})$$

6. Challenges and Future Perspectives

- Precise determination of quark masses and potential parameters.
- Inclusion of relativistic corrections, especially for charm quarks.
- Extension of HH to tetraquark and pentaquark systems.
- Comparison with lattice QCD predictions and experimental data from LHCb and Belle II.

7. Conclusion

The Hyperspherical Harmonics method, combined with the Constituent Quark Model, provides a reliable and systematic framework to study heavy baryons. It allows precise predictions of mass spectra, internal structures, and decay dynamics. Future developments in HH techniques and experimental validations will further advance the understanding of heavy quark dynamics.

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